

Hermite, sviluppi (it.scienza.matematica)

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Problema

Trovare $f(x) \geq 0$, con $\text{area}=1$ e tale che

$$\int_0^{\infty} f(h+x) f(x) dx = e^{-h^2}$$

Sviluppo di $f(x)=\text{Exp}(-x^2/2)*g(x)$ in serie di Hermite

Provo a sviluppare solo con gli H di una certa parita'? Qui provo solo con le pari. $q[k]$ e' quindi il coefficiente di $H(2k)[x]$ nello sviluppo

`In[195]:=`

```
Clear[q];
tot = 2;
Q[x_] := Sum[q[k] * H[2 * k, x], {k, 0, tot}] // Evaluate
Q[x]
```

`Out[198]=`

```
q[0] + (-2 + 4 x^2) q[1] + (12 - 48 x^2 + 16 x^4) q[2]
```

`In[199]:=`

Condizione di area =1

`In[200]:=`

```
CondArea = {Integrate[Exp[-x^2/2] * Q[x], {x, 0, Infinity}] - 1}
```

`Out[200]=`

```
{-1 + Sqrt[Pi/2] (q[0] + 2 (q[1] + 6 q[2]))}
```

Calcolo correlazione

`In[201]:=`

```
r = Q[x + h] * Q[x] // Expand // Simplify;
```

```
In[202]:=
r = Integrate[ Exp[-x^2 - h*x] * r, {x, 0, Infinity}, Assumptions -> h > 0]
```

```
Out[202]=

$$\frac{1}{2} e^{\frac{h^2}{4}} \sqrt{\pi} \operatorname{Erfc}\left[\frac{h}{2}\right] \left( q[0]^2 + (8 - 8h^2 + h^4) q[1]^2 + 2h^2 (48 - 16h^2 + h^4) q[1] q[2] + \right. \\ \left. (384 + h^2 (-8 + h^2) (96 - 24h^2 + h^4)) q[2]^2 + 2h^2 q[0] (q[1] + h^2 q[2]) \right) + \\ h \left( -(-2 + h^2) q[1]^2 - 2(-12 - 2h^2 + h^4) q[1] q[2] - (-120 + 172h^2 - 34h^4 + h^6) q[2]^2 + \right. \\ \left. 2q[0] (q[1] + (-6 + 7h^2) q[2]) \right)$$

```

■ Sviluppo in potenze di h della correlazione

```
In[203]:=
coeff1 = CoefficientList[r + O[h]^tot, h] // Expand;
coeff1
```

```
Out[204]=

$$\left\{ \frac{1}{2} \sqrt{\pi} q[0]^2 + 4 \sqrt{\pi} q[1]^2 + 192 \sqrt{\pi} q[2]^2, \right. \\ \left. -\frac{1}{2} q[0]^2 + 2q[0] q[1] - 2q[1]^2 - 12q[0] q[2] + 24q[1] q[2] - 72q[2]^2 \right\}$$

```

```
In[205]:=
n = Length[coeff1]
```

```
Out[205]=
2
```

■ Sviluppo in potenze di h di exp(-h^2/2)

```
In[206]:=
coeff2 = Take[ CoefficientList[Exp[-h^2/2] + O[h]^5, h], n]
```

```
Out[206]=
{1, 0}
```

```
In[207]:=
Condsviluppi = coeff1 - coeff2
```

```
Out[207]=

$$\left\{ -1 + \frac{1}{2} \sqrt{\pi} q[0]^2 + 4 \sqrt{\pi} q[1]^2 + 192 \sqrt{\pi} q[2]^2, \right. \\ \left. -\frac{1}{2} q[0]^2 + 2q[0] q[1] - 2q[1]^2 - 12q[0] q[2] + 24q[1] q[2] - 72q[2]^2 \right\}$$

```

Risolvo il sistema ...

```
In[208]:=
Clear[q];
x = Table[q[i], {i, 0, tot}]
```

```
Out[209]=
{q[0], q[1], q[2]}
```

```

In[210]:=
  CondReale = Table[ Im[q[i]], {i, 0, tot}]

Out[210]=
  {Im[q[0]], Im[q[1]], Im[q[2]]}

In[211]:=
  sistema = Union[CondArea, CondSviluppi, CondReale]

Out[211]=
  {Im[q[0]], Im[q[1]], Im[q[2]],
   - $\frac{1}{2}$  q[0]2 + 2 q[0] q[1] - 2 q[1]2 - 12 q[0] q[2] + 24 q[1] q[2] - 72 q[2]2,
   -1 +  $\frac{1}{2}$   $\sqrt{\pi}$  q[0]2 + 4  $\sqrt{\pi}$  q[1]2 + 192  $\sqrt{\pi}$  q[2]2, -1 +  $\sqrt{\frac{\pi}{2}}$  (q[0] + 2 (q[1] + 6 q[2]))}

In[212]:=
  sol = NSolve[ sistema == 0, x]

InverseFunction::ifun :
  Inverse functions are being used. Values may be lost for multivalued inverses. More...

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General::stop : Further output of InverseFunction::ifun will be suppressed during this calculation. More...

Solve::incnst : Inconsistent or redundant transcendental
  equation. After reduction, the bad equation is -q[0] + 2 q[1] - 12 q[2] == 0. More...

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  equation. After reduction, the bad equation is -2 + 4  $\sqrt{2\pi}$  q[1] == 0. More...

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  reduction, the bad equation is -2 + 12  $\sqrt{\pi}$  q[1]2 - 48  $\sqrt{\pi}$  q[1] q[2] + 528  $\sqrt{\pi}$  q[2]2 == 0. More...

General::stop : Further output of Solve::incnst will be suppressed during this calculation. More...

Out[212]=
  {{q[2] → -0.0271961, q[1] → 0.199471, q[0] → 0.725295},
   {q[2] → -0.0271961, q[1] → 0.199471, q[0] → 0.725295},
   {q[2] → 0.0453298, q[1] → 0.199471, q[0] → -0.145015},
   {q[2] → 0.0453298, q[1] → 0.199471, q[0] → -0.145015}}

```

La soluzione

Soluzione scelta:

```

In[213]:=
  s = 1; sol[[s]]

Out[213]=
  {q[2] → -0.0271961, q[1] → 0.199471, q[0] → 0.725295}

```

■ Serie di potenze:

```
In[214]:=
Clear[f]
f[x_, s_: 1] := ((Q[x] /. sol[[s]]) // Expand) * Exp[-x^2 / 2]
f[x, s]
```

```
Out[216]=

$$e^{-\frac{x^2}{2}} (5.55112 \times 10^{-17} + 2.1033 x^2 - 0.435137 x^4)$$

```

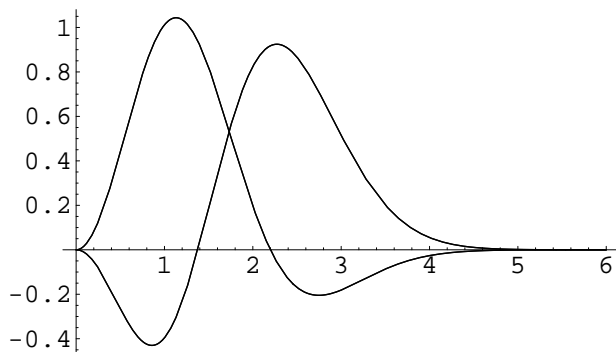
■ Sviluppo in H:

```
In[217]:=
Sum[q[k] * HH2k[x], {k, 0, n}] /. sol[[s]]
```

```
Out[217]=
0.725295 HH0[x] + 0.199471 HH2[x] - 0.0271961 HH4[x]
```

Verifiche:

```
In[218]:=
Plot[Table[f[x, k], {k, 1, Length[sol]}] // Evaluate, {x, 0, 6}]
```



```
Out[218]=
- Graphics -
```

```
In[219]:=
Integrate[f[x, s], {x, 0, Infinity}]
```

```
Integrate::gener : Unable to check convergence. More...
```

```
Out[219]=
1.
```

```
In[220]:= Integrate[ f[x+h, s] * f[x, s], {x, 0, Infinity}] + O[h]^8 // N
```

```
Integrate::gener : Unable to check convergence. More...
```

```
Out[220]= 1. - 2.22045 × 10-16 (h + 0.) - 1.2406 (h + 0.)2 + 1.11022 × 10-16 (h + 0.)3 + 0.621822 (h + 0.)4 -  
0.0737309 (h + 0.)5 - 0.149881 (h + 0.)6 + 0.0297824 (h + 0.)7 + O[h + 0.]8
```

Avvisami quando il calcolo e' finito

```
In[221]:= Run["/home/michele/cmd/alarm"]
```

```
Out[221]= -1
```